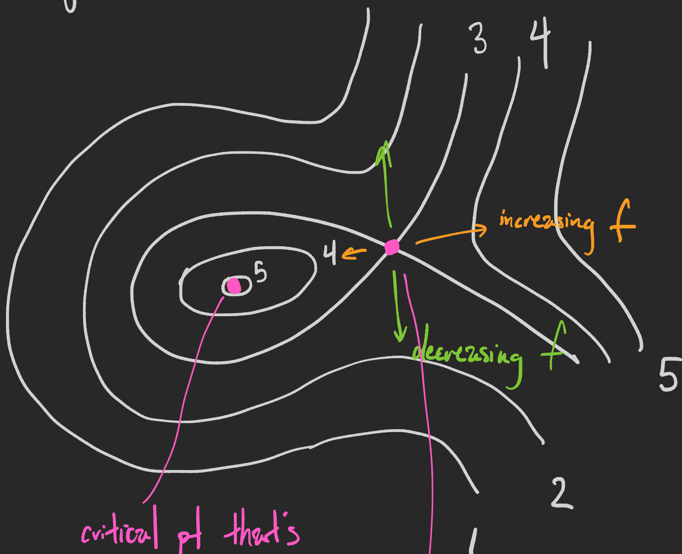
$f(x,y)$ a function. Some of its level sets:



critical pt that's a local max

Impossible to draw a nonzero vec. perpendicular to level set @ this point, so ∇f is either $\vec{0}$ here, or DNE.

(so it is a critical pt.)

i.e. if a level set ever looks like

this must be a critical point.

Let $f(x, y) = x^2 y$. Suppose we want to find the tangent plane to the graph of this function when $(x, y) = (3, 0)$, and that we want to use the $\nabla F(\mathbf{r}_0) \cdot (\mathbf{r} - \mathbf{r}_0) = 0$ formula. What is an appropriate choice of F?

What do we use as $\mathbf{r}_0$?

Recall, the graph of $f(x,y)$ is the surface defined by

$$z = f(x,y)$$

e.g.

$$z = x^2 y. \quad (*)$$

Recall the plane formula:

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

a normal vec

$\langle x, y, z \rangle$

any pt. in the plane

Rearrange $(*)$ as $= \text{constant}$

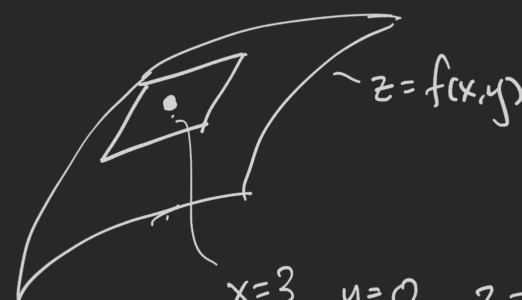
e.g.

$$x^2 y - z = 0$$

$$x^2 y / z = 1$$

In our situation, $z = 0$ @ pt in question, so second is no good (but at a pt. where $z \neq 0$ this would be fine too).

So let $F(x,y,z) = x^2 y - z$.



$x = 3, \quad y = 0, \quad z = f(3,0) = 0.$

$$\nabla F(\vec{r}_0) = \nabla F(3,0,0)$$

$$= \langle 0, 9, -1 \rangle$$

So tan. plane eq is

$$\langle 0,9,-1\rangle \cdot \left(\langle x,y,z\rangle - \langle 3,0,0\rangle\right) = 0$$

$$9y - z = 0$$

or $z = 9y$.

Consider the plane $z = 9y$. Which of the following lines is *perpendicular* to this plane?

$x = 2, y = -3 + 9t, z = -2 + t$

$\mathbf{r}(t) = \langle 0, -2, 7\rangle + t\langle 0, 18, -2\rangle$

$$\frac{x-7}{3} = y - 2 = \frac{z+1}{9}$$

None of the above

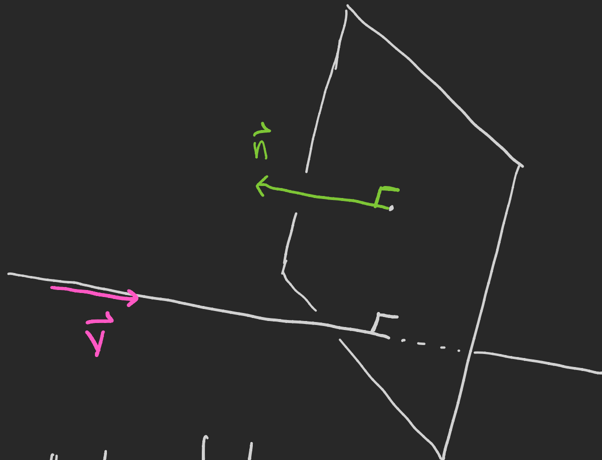
Consider the plane $z = 9y$. Which of the following lines is *parallel* to this plane?

$x = 2,\ y = -3 + 9t,\ z = -2 + t$

$\mathbf{r}(t) = \langle 0, -2, 7\rangle + t\langle 0, 18, -2\rangle$

$$\frac{x-7}{3} = y - 2 = \frac{z+1}{9}$$

None of the above

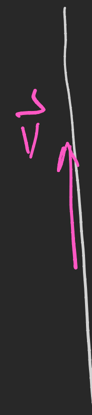


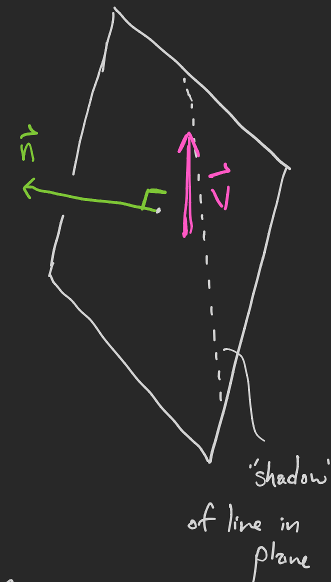
So the line and plane are perpendicular when $\vec{n}$ and $\vec{v}$ are parallel.

$$9y - z = 0$$

$$0x + 9y + (-1)z = 0$$

So $\vec{n} = \langle 0, 9, -1 \rangle$ for example.





So the line and plane are parallel when

$$\vec{n} \cdot \vec{v} = 0.$$

$$\frac{x-7}{3} = y-2 = \frac{z+1}{9} = t$$

$x = 3t+7 \quad y = t+2 \quad z = 9t-1.$

$\vec{v} = \langle 3, 1, 9 \rangle$